## 245. Theoretical Studies of the Macrocyclic Pigments. Part I. The Structure of Bacteriochlorophyll.

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Molecular-orbital calculations have been performed on chlorin and a tetrahydroporphin of $C_{2 \pi}$ symmetry. It is found that the predicted centres of gravity for the longest-wavelength transitions lie in the order $\mathrm{C}_{2 \mathrm{v}}$ tetrahydroporphin $<$ porphin $<$ chlorin $<\mathrm{D}_{2 \mathrm{~h}}$ tetrahydroporphin. This is in agreement with spectra in the tetraphenylporphin series, where the porphin and the chlorin lie between two tetrahydro-compounds of unknown structure. It follows that bacteriochlorophyll is related to the tetrahydroporphin of $\mathrm{D}_{2 \mathrm{~h}}$ symmetry in which the two pairs of hydrogen atoms are located on opposite pyrrole rings. This effect of symmetry is also observed in the polycyclic aromatic hydrocarbons related to coronene.

There are only two structural features in which bacteriochlorophyll differs from chloro-phyll-a. This close relation is exemplified by the conversion of the former into 2 -acetyl2 -devinylphæophorbide- $a$ which has been synthesised from chlorin- $e_{6} \cdot{ }^{1}$ The degradation is effected by the removal of the magnesium atom, the phytyl residue, and the (? two) extra hydrogen atoms. ${ }^{2}$ Bacteriochlorophyll has an acetyl group at position 2 and differs in its state of hydrogenation. The preservation of long-wavelength absorption (ca. $7500 \AA$ )

[^0]${ }^{2}$ Fischer, Lambrecht, and Mittenzwei, Z. physiol. Chemie, 1938, 253, 1.
and the ease of dehydrogenation ${ }^{3}$ suggest that the two extra hydrogen atoms are located on one of the three pairs of peripheral or $\beta$-positions of ring I, II, or III (see structure I).* Thus, apart from the stereochemistry of the various asymmetric centres, the only outstanding problem regarding the structure of bacteriochlorophy! 1 is the decision between these three pairs of positions.

Mittenzwei ${ }^{3}$ has suggested that ring II carries the extra hydrogen atoms and that bacteriochlorophyll has the structure (I). He has shown that oxidative degradation of bacteriochlorophyll derivatives yields small quantities of an oil which was possibly ethylmethylsuccinic anhydride. As this compound is not produced when chlorophyll derivatives are similarly degraded Mittenzwei claims that it must have arisen from a hydrogenated ring II. Seely ${ }^{5}$ recently criticised the significance of these findings on the grounds that the failure of chlorophyll derivatives to yield ethylmethylsuccinic anhydride does not exclude the possibility of its having arisen from ring IV by decarboxylation before degradation of the bacteriochlorophyll derivatives. Seely suggested that the extra hydrogen atoms may be in either ring I or ring III, probably the latter.


(III)

(II)

(IV)

We agree with Seely that Mittenzwei's proof of structure is by no means rigid. We now provide, by the application of the empirical L.C.A.O. M.O. method, some support for the correctness of structure (I). $\dagger$ Our studies suggest marked spectral differences for the two tetrahydroporphin models (II) and (III) which, if they exist, would allow an unambiguous

* There is now conclusive evidence that ring IV in chlorophyll has a 3:4-dihydropyrrole structure. ${ }^{4}$ $\dagger$ Professor Longuet-Higgins (personal communication) has stressed the fact that too much reliance should not be placed on simple M.O. methods when applied to the calculation of transition energies and that more refined calculations using self-consistent orbitals would be better adapted for this purpose. We are undertaking a more detailed treatment.

[^1]decision between the two types. This effect is dependent on symmetry and may be quite general for large conjugated loops. Analogous observations in other systems of known constitution tend to confirm the theoretical findings. Longuet-Higgins, Rector, and Platt ${ }^{6}$ accounted for the bathochromic shift which accompanies hydrogenation of the porphin nucleus to the bacteriochlorophyll level by assuming structure (II) for the latter and used the simple M.O. method including overlap. We have extended the calculations to the $\mathrm{C}_{2 \mathrm{v}}$ model, (III), as well as to chlorin, the dihydro-level. We find that the structure (III) is not associated with a bathochromic shift.

The $\pi$-electron system of porphin itself has the symmetry of the point group $\mathrm{D}_{4 \mathrm{~h}}$. Like many conjugated systems with $\mathrm{D}_{3 \mathrm{~h}}$ or higher symmetries the low-lying $\pi$-electron transitions are to an unoccupied degenerate M.O. In consequence the spectrum predicted from simple M.O. theory might be expected to be altered, perhaps considerably, by configuration interaction. ${ }^{7}$ Although Longuet-Higgins et al. ${ }^{6}$ originally suggested that this alters the intensities of porphin rather than the predicted frequencies,* we have thought it better to compare the predicted spectra of the two tetrahydro-models with that of chlorin as none of these structures has degenerate levels and configuration interaction will be less serious than in porphin.

In applying M.O. methods to porphin molecules two rather serious approximations, besides those implicit in the general method, have to be made. These concern the value of $\alpha_{N}$, the Coulomb integral for nitrogen, and the variation of $\alpha_{i}$, the Coulomb integral of the $i$ 'th atom with its charge density $\dot{q}_{i}$. It is convenient to express $\alpha_{N}$ in terms of $\alpha_{0}$ by means of equation (1) in which $\beta$ is the resonance integral for a carbon-carbon double bond :

$$
\begin{equation*}
\alpha_{N}=\alpha_{\mathrm{C}}+\beta \delta \tag{1}
\end{equation*}
$$

Values of $\delta$ ranging from 0.6 to 2.0 are found in the literature. ${ }^{8}$ We have attempted to avoid the necessity of selecting one value for $\delta$ by examining an extreme range of values (viz., 0-2.0). This is readily done by using the method of Coulson and Longuet-Higgins ${ }^{9}$ in which the change in $\alpha$ is treated as a perturbation. $\delta \varepsilon_{r}$, the first order perturbation energy for the $r$ 'th M.O., is given by equation (2)

$$
\begin{equation*}
\delta \varepsilon_{r}=\sum c^{2}{ }_{i r} . \delta \alpha_{i} \tag{2}
\end{equation*}
$$

where $c_{i r}$ is the coefficient of the atom $i$ in the $r^{\prime}$ th M.O. Longuet-Higgins et al. ${ }^{6}$ have already applied this correction, with $\delta=1$, to porphin and $\mathrm{D}_{2 \mathrm{~h}}$ tetrahydroporphin (II). We have also calculated sets of data in which $\alpha_{N}$ has been varied with $q$. The results for porphin, chlorin, and the two tetrahydroporphins have been collected in Table l. Ońly the first transitions have been considered. The energy values have also been converted into wavelengths by use of conversion factors obtained by assuming the energy for the first transition of chlorin to correspond to $6375 \AA$, the experimentally determined position of the longestwavelength band in chlorin, the structure of which has been rigidly proved. ${ }^{10}$ The value of $\beta$ used to convert the energy into frequency is also included in Table 1 and may be compared with the values for $\beta$ of $-23,000$ and $-30,000 \mathrm{~cm} .^{-1}$ which best reproduce the experimental values in the polycyclic aromatic hydrocarbon series and the polyenes respectively. ${ }^{7}$

Fortunately we have certain empirical criteria which we may legitimately use to limit the choice of $\delta$ and its variation with charge density. If we select chlorin as a reference compound, the calculated data for the first transition must fulfil two conditions : (1) At least one of the two tetrahydro-structures (II) and (III) must have its first transition of lower energy than the first transition of chlorin. This follows from the fact that bacteriochlorophylls always have their longest-wavelength band some $1000 \AA$ to longer wavelengths

[^2]than that of the corresponding chlorins. (2) The first transition of a chlorin is always observed to be of lower energy than that of the corresponding porphin. The M.O. method with configuration interaction included should predict this sequence. But since configuration interaction tends to reduce the energies of first transitions and since its effect will be greater in porphin than in chlorin, it follows that simple M.O. theory without configuration interaction should predict the same sequence.

Two important conclusions may now be drawn from Table 1. First, the theory predicts that even for the extreme range of $\delta$ considered the tetrahydroporphin (II) of symmetry $\mathrm{D}_{2 \mathrm{~h}}$ always has its longest-wavelength band at longer wavelengths than that (III) of $\mathrm{C}_{2 \mathrm{v}}$ symmetry. Secondly, if the choice of $\delta$ and its variation with $q$ is limited to those values which fulfil the two foregoing empirical criteria (i.e., for values of $\delta$ between about 0.7 and $1 \cdot 6$ ), the predicted long-wavelength absorption of the $D_{2 h}$ model (II) lies above that of chlorin, while that of the $\mathrm{C}_{2 \mathrm{r}}$ compound lies below that of chlorin.

Table 1. The energies, wavelengths, and symmetry types predicted for the longestwavelength bands of porphin and its hydro-derivatives.

|  | $\delta$ : 0 |  | $0 \cdot 6{ }^{\text {a }}$ |  | 0.8 |  | $1.0{ }^{\text {a }}$ |  | 1.2 |  | $1.4{ }^{\text {a }}$ |  | $2 \cdot 0$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Porphin | $\Delta y / \beta 0 \cdot 391$ | 0.543 | $0 \cdot 491$ | $0 \cdot 435$ | 0.588 | $\overparen{0.627}$ | $0 \cdot 583$ | $0 \cdot 534$ | $0 \cdot 665$ | $0 \cdot 641$ | $0 \cdot 662$ | $0 \cdot 620$ | $0 \cdot 529$ |
|  | $\lambda(\AA) 6300$ | 5700 | 5300 | 4760 | 5590 | 5520 | 5180 | 4720 | 5470 | 5890 | 5070 | 4690 | 9040 |
|  | $\stackrel{\mathrm{A}_{2 u^{-}}{ }_{\mathrm{g}}}{ }$ |  | $\begin{gathered} \mathrm{A}_{2 \mathrm{u}}- \\ \mathrm{E}_{\mathrm{g}} \end{gathered}$ |  | $\stackrel{\mathrm{A}_{2 \mathrm{n}}-}{\mathrm{E}_{\mathrm{g}}}$ |  | $\begin{gathered} \mathrm{A}_{2 \mathrm{nn}} \\ \mathrm{E}_{\mathrm{g}} \end{gathered}$ |  | $\underset{\mathrm{E}_{\mathrm{g}}}{\mathrm{~A}_{\mathrm{E}^{-}}}$ | $\underset{\mathrm{E}_{\mathrm{g}}}{\mathrm{~A}_{1}}$ | $\underset{\mathrm{E}_{\mathrm{g}}}{\mathrm{~A}_{\mathrm{gn}}-}$ | $\begin{gathered} \mathrm{A}_{2 \mathrm{u}}- \\ \mathrm{E}_{\mathrm{g}} \end{gathered}$ | $\stackrel{\mathrm{A}_{\mathrm{E}_{\mathrm{g}}}}{ }$ |
| Chlorin | $\Delta y / \beta \quad 0.386$ | $0 \cdot 486$ | $0 \cdot 408$ | $0 \cdot 325$ | 0.516 | 0.542 | 0.471 | $0 \cdot 395$ | 0.568 | 0.592 | 0.526 | $0 \cdot 456$ | 0.756 |
|  | $\lambda(\AA) 6375$ | 6375 | 6375 | 6375 | 6375 | 6375 | 6375 | 6375 | 6375 | 6375 | 6375 | 6375 | 6375 |
|  | $\begin{gathered} \mathrm{B}_{1}- \\ \mathrm{B}_{1} \end{gathered}$ |  | $\begin{gathered} \mathrm{B}_{1}- \\ \mathrm{B}_{1} \end{gathered}$ |  | $\begin{aligned} & \mathrm{B}_{1}- \\ & \mathrm{B}_{1} \end{aligned}$ |  | $\begin{gathered} \mathrm{B}_{1}- \\ \mathrm{B}_{1} \end{gathered}$ |  | $\begin{gathered} \mathrm{B}_{1}- \\ \mathrm{B}_{1} \end{gathered}$ |  | $\mathrm{B}_{1}-$ $\mathrm{B}_{1}$ |  | $\xrightarrow{\mathrm{B}_{1}-}$ |
| $\mathrm{D}_{2 \mathrm{~h}}$ Tetrahydroporphin | $\Delta y / \beta 0 \cdot 380$ | 0.514 | $0 \cdot 446$ | $0 \cdot 383$ | $0 \cdot 497$ | $0 \cdot 449$ | $0 \cdot 468$ | $0 \cdot 464$ | $0 \cdot 402$ | $0 \cdot 356$ | $0 \cdot 374$ | $0 \cdot 393$ | $0 \cdot 225$ |
|  | $\lambda(\AA) 6480$ | 6020 | 5840 | 5410 | 6620 | 7700 | 6380 | 5430 | 9010 | 10600 | 8970 | 7400 | 21200 |
|  | $\mathrm{B}_{1 u^{-}}$ |  | $\mathrm{B}_{14}{ }^{-}$ |  | $\mathrm{A}_{\mathrm{u}}{ }^{-}$ | $\mathrm{A}_{\mathrm{u}}{ }^{-}$ | $\mathrm{A}_{\mathrm{u}}{ }^{-}$ | $\mathrm{B}_{1 u^{-}}$ | $\mathrm{A}_{\mathrm{u}}{ }^{-}$ |  | $\mathrm{A}_{\mathrm{u}}{ }^{-}$ |  | $\mathrm{A}_{\mathrm{u}}{ }^{-}$ |
|  | $\mathrm{B}_{3 g}$ |  | $\mathrm{B}_{3 \mathrm{~g}}$ |  | $\mathrm{B}_{3 \mathrm{~g}}$ | $\mathrm{B}_{35}$ | $\mathrm{B}_{35}$ | $\mathrm{B}_{3 \mathrm{~g}}$ | $\mathrm{B}_{35}$ |  | $\mathrm{B}_{3 \mathrm{sg}}$ |  | $\mathrm{B}_{38}$ |



| hydroporphin $\gamma(\AA) 5110$ | 4780 | 4330 | 3760 | 4920 | 5410 | 4500 | 3840 | 845 | 20 | 80 | 5280 | 09030 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }_{\text {B1 }} \mathrm{B}_{1}$ |  | $\mathrm{B}_{1} \mathrm{~B}_{1}$ |  | ${ }_{\text {A }}{ }_{\text {B }}$ | ${ }_{\text {A }} \mathrm{A}_{\mathrm{B}_{1}}$ | ${ }_{\text {A }} \mathrm{A}_{2}$ | $\stackrel{\mathrm{B}_{1}-1}{\mathrm{~B}_{1}}$ | $\mathrm{A}_{\mathrm{B}_{1}}$ | ${ }_{\text {A }}{ }_{\text {A }}$ | $\stackrel{\mathrm{B}_{1}}{\mathrm{~B}_{1}}$ | ${ }_{1}^{\mathrm{B}_{1}} \mathrm{~B}_{1}$ | $\mathrm{A}_{2}{ }_{\text {B }}$ |

$\begin{array}{lllllllllllllll}-10^{4} \beta\left(\mathrm{~cm} .^{-1}\right) & 4.06 & 3.23 & 3.84 & 4.83 & 3.04 & 2.89 & 3.35 & 3.97 & 2.76 & 2.65 & 2.98 & 3.44 & 2.09\end{array}$ ${ }^{a}$ The first set refers to $\delta$; the second to $(\delta+1-q)$, and the third to $(\delta+2-2 q)$.

It is fortunate that there exists one series in which all four compounds and their spectra are probably known, viz., the mesotetraphenyl series. Dorough and Miller ${ }^{11}$ have prepared the two tetrahydro-pigments and have established their higher state of hydrogenation by dehydrogenation to the chlorin.* Results for the longest-wavelength band of these compounds are given in Table 2, together with similar values for other incomplete series.

Table 2. Wavelengths $(\AA)$ of the longest-wavelength bands of some porphin derivatives.
meso Tetraphenylporphin, ${ }^{a} 6480$.
mesoTetraphenylchlorin, ${ }^{a, b} 6550$.
Tetrahydrotetraphenylporphin compound $\alpha,{ }^{b} 7500$. Tetrahydrotetraphenylporphin compound $\beta,{ }^{b} 5910$. Porphin, ${ }^{c} 6160$.
${ }^{a}$ Ball, Dorough, and Calvin, J. Amer. Chem. Soc., 1946, 68, 2278; ${ }^{b}$ Dorough and Miller, ibid., 1952, 74, 6106; ${ }^{c}$ Eisner and Linstead, J., 1955, 3742; ${ }^{\text {a }}$ Fischer, Lautsch, and Lin, Annalen, 1938, 534, 1; ' Fischer and Riedmair, ibid., 1933, 505, 87; ' Weigl, J. Amer. Chem. Soc., 1953, ry, 999.

It is seen in the mesotetraphenyl series that the two tetrahydro-pigments are in fact in the predicted positions relative to the chlorin. The remarkable spectral difference predicted

[^3]11 Dorough and Miller, J. Amer. Chem. Soc., 1952, 74, 6106.
for the two types of symmetry is therefore experimentally established and we can with confidence assign structure (I) to bacteriochlorophyll.

Our results seem quite general for the symmetry changes involved, for the predictions based on calculations in the porphin series may be successfully applied to the polycyclic aromatic hydrocarbon series. Longuet-Higgins et al. ${ }^{6}$ have already noted the parallelism between the series porphin, chlorin, and $\mathrm{D}_{2 \mathrm{~h}}$ tetrahydroporphin, and the series coronene, $1: 2$-benzoperylene, and perylene. We now find that $\mathrm{C}_{2 \mathrm{v}}$ tetrahydroporphin and 1:2benzopyrene may be included in the respective series (cf. Table 3 ).

Table 3. Comparison of the longest-wavelength $(\AA)$ bands of meso-tetraphenylporphins and polycyclic aromatic hydrocarbons.*

$\mathrm{D}_{4 \mathrm{~h}}$
Porphin
6480






* The data for the hydrocarbons refer to the $p$-bands (Clar, " Aromatische Kohlenwasserstoffe," Springer, Berlin, 1952.

The results described here provide an example of the manner in which the methods of theoretical chemistry, in spite of their many approximations, can be employed for solving structural problems. We have purposely avoided the choice of a single value for $\delta$. It is gratifying to note, however, that the experimental results are best reproduced by $\delta \sim \mathbf{l}$ which is the value generally assumed for this parameter. ${ }^{12}$ Further the values of $\beta$ in this region agree satisfactorily with the value usually assumed (see p. 1174). If the value of $\delta$ is approximately unity, the $7500 \AA$ band of $D_{2 \mathrm{~h}}$ tetrahydroporphin is due to the $\mathrm{A}_{\mathrm{u}}-\mathrm{B}_{3 \mathrm{~g}}$ transition and is polarised parallel to the long axis. The method of correction for the variation of $\alpha$ with $q_{i}$ is, at best, a very crude approximation. We have used the $q_{i}$ 's derived for the all-carbon model and have only considered the variation of $\delta$ for the four nitrogen atoms. Although the errors involved cannot alter the present conclusions a more satisfactory treatment is necessary for the predictions of other properties of these molecules.

## Calculations

We have corrected all M.O. energies for overlap by the usual method ${ }^{13}$ in which

$$
\begin{aligned}
& y_{r}=4 x_{r} /\left(4+x_{r}\right) \\
& x_{r}=\left(\varepsilon_{r}-\alpha\right) / \beta
\end{aligned}
$$

where
$\varepsilon_{r}$ being the enery of the $r^{\prime}$ th M.O. This is an approximation to the more accurate correction used by Wheland ${ }^{14}$ when the coulomb integrals vary.

As no account of spin interaction is included the predicted transition energies refer to the centres of gravity of the singlet-singlet and singlet-triplet transitions. We have made no explicit correction for the effect of the two pyrrolic hydrogen atoms. Their effect is probably

[^4]small and acts so as to reduce $\alpha_{N}$. It is therefore absorbed by the range of values considered for the variation of $\delta_{i}$ with $q_{i}$.

Porphin and $\mathrm{D}_{2 \mathrm{~h}}$ Tetrahydroporphin.-The M.O. energies and $c_{i}$ 's have been calculated by Longuet-Higgins, Rector, and Platt. ${ }^{6}$ We have used these values to calculate the additional data for the various $\delta$ 's.

Chlorin.-A recent theoretical treatment of this compound gives secular equations which include explicit corrections for the pyrrolic hydrogen atoms but take no account of the variation of $\alpha_{i}$ with $q_{i} .{ }^{15}$ Chlorin belongs to the symmetry group $\mathrm{C}_{2 \mathrm{v}}$, the M.O. wave functions being bases of the irreducible representations $B_{1}$ and $A_{2}$. The roots of the secular equations and the corresponding coefficients are given in Table 4. Table 5 lists the charge densities at the various positions.

TABLE 4. Values of $x_{\mathrm{r}}$ and $c_{\mathrm{ir}}$ for chlorin.
Representation $A_{2}$.

|  |  |  |  |  |  |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $x_{r}$ | -2.0655 | -1.9197 | -1.6180 | -0.9247 | -0.4680 | 0.4472 | 0.6180 | 1.0985 | 1.5805 | 2.2517 |
| $c_{1}$ | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| $c_{2}$ | 0.0605 | 0.1491 | 0.0000 | 0.4176 | 0.1700 | 0.3851 | 0.0000 | 0.2844 | 0.1885 | 0.0782 |
| $c_{3}$ | -0.1250 | -0.2863 | 0.0000 | -0.3862 | -0.0796 | 0.1722 | 0.0000 | 0.3124 | 0.2979 | 0.1760 |
| $c_{4}$ | 0.1976 | 0.4005 | 0.0000 | -0.0605 | -0.1328 | -0.3081 | 0.0000 | 0.0588 | 0.2824 | 0.3181 |
| $c_{5}$ | -0.0174 | -0.3175 | -0.1730 | 0.2734 | -0.1030 | -0.0897 | -0.3897 | -0.2042 | 0.1294 | 0.2554 |
| $c_{6}$ | -0.1618 | 0.2091 | 0.2799 | -0.1923 | 0.1810 | 0.2679 | -0.2408 | -0.2830 | -0.0778 | 0.2570 |
| $c_{7}$ | 0.3515 | -0.0839 | -0.2799 | -0.0956 | 0.0183 | 0.2096 | 0.2409 | -0.1067 | -0.2524 | 0.3233 |
| $c_{8}$ | -0.2984 | 0.1168 | 0.0000 | 0.1119 | -0.4342 | 0.0461 | 0.0000 | 0.2095 | -0.3401 | 0.1862 |
| $c_{9}$ | 0.2648 | -0.1404 | 0.2799 | -0.0079 | 0.1850 | -0.1890 | -0.2409 | 0.3368 | -0.2851 | 0.0958 |
| $c_{10}$ | -0.2485 | 0.1527 | -0.4529 | -0.1046 | 0.3476 | -0.1306 | -0.1489 | 0.1605 | -0.1105 | 0.0295 |
| $c_{20}$ | -0.2659 | -0.1649 | 0.1730 | 0.1688 | 0.2447 | -0.2203 | 0.3897 | -0.0437 | 0.0190 | 0.2849 |
| $c_{21}$ | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |


| Representation $B_{1}$. |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $x_{r}$ | $-3 \cdot 1447$ | -2.0306 | -1.6180 | $-1.5167$ | -0.3519 | $0 \cdot 0000$ |
| $c_{1}$ | $0 \cdot 1278$ | $0 \cdot 2877$ | $0 \cdot 0000$ | $0 \cdot 5074$ | $0 \cdot 3327$ | $0 \cdot 3780$ |
| $c_{2}$ | -0.1370 | -0.2921 | $0 \cdot 0000$ | $-0.3848$ | $-0.0586$ | $0 \cdot 0000$ |
| $c_{3}$ | $0 \cdot 1661$ | $0 \cdot 3054$ | $0 \cdot 0000$ | $0 \cdot 0762$ | $-0.3122$ | $-0.3780$ |
| $c_{4}$ | -0.2192 | -0.3281 | $0 \cdot 0000$ | $0 \cdot 2692$ | $0 \cdot 1684$ | $0 \cdot 0000$ |
| $c_{5}$ | $0 \cdot 0256$ | $0 \cdot 2395$ | -0.2021 | -0.2648 | $0 \cdot 2578$ | $0 \cdot 0000$ |
| $c_{6}$ | $0 \cdot 1643$ | -0.1582 | $0 \cdot 3271$ | $0 \cdot 1325$ | $-0.2591$ | $0 \cdot 0000$ |
| $c_{7}$ | $-0.3268$ | 0.0818 | -0.3271 | $0 \cdot 0639$ | $-0 \cdot 1666$ | $0 \cdot 0000$ |
| $c_{8}$ | $0 \cdot 2820$ | -0.1291 | $0 \cdot 0000$ | $-0.0098$ | $0 \cdot 3226$ | $-0.3780$ |
| $c_{9}$ | -0.2780 | $0 \cdot 1805$ | $0 \cdot 3271$ | -0.0491 | $0 \cdot 0531$ | $0 \cdot 0000$ |
| $c_{10}$ | 0.0884 | $-0.0596$ | -0.1249 | 0.0195 | $-0.0393$ | $0 \cdot 0000$ |
| $c_{20}$ | $0 \cdot 2546$ | 0.1213 | $0 \cdot 2021$ | $-0.2196$ | $-0.0049$ | $0 \cdot 3780$ |
| $c_{21}$ | $0 \cdot 2593$ | -0.1777 | $-0.4043$ | $0 \cdot 0647$ | -0.3020 | $0 \cdot 0000$ |
| $x_{r}$ | $0 \cdot 6180$ | 0.7183 | $1 \cdot 0965$ | $1 \cdot 7365$ | $2 \cdot 1314$ | $2 \cdot 3612$ |
| $c_{1}$ | $0 \cdot 0000$ | $0 \cdot 1995$ | $0 \cdot 3315$ | $0 \cdot 4795$ | $0 \cdot 1695$ | $0 \cdot 0683$ |
| $c_{3}$ | $0 \cdot 0000$ | 0.0717 | $0 \cdot 1818$ | $0 \cdot 4163$ | $0 \cdot 1806$ | $0 \cdot 0806$ |
| $c_{3}$ | $0 \cdot 0000$ | $-0.1481$ | $-0.1322$ | $0 \cdot 2434$ | 0.2155 | $0 \cdot 1221$ |
| $c_{4}$ | $0 \cdot 0000$ | $-0.1780$ | $-0.3267$ | $0 \cdot 0064$ | $0 \cdot 2786$ | $0 \cdot 2077$ |
| $c_{5}$ | $-0.2538$ | $0 \cdot 2760$ | $-0.2151$ | -0.1063 | 0-1977 | 0-1661 |
| $c_{6}$ | $-0.1568$ | $0 \cdot 3763$ | 0.0909 | -0.1909 | $0 \cdot 1427$ | $0 \cdot 1846$ |
| $c_{7}$ | $0 \cdot 1568$ | $-0.0057$ | $0 \cdot 3148$ | $-0.2253$ | $0 \cdot 1065$ | $0 \cdot 2697$ |
| $c_{8}$ | $0 \cdot 0000$ | $-0.1246$ | $0 \cdot 2652$ | $-0.0743$ | $-0.0964$ | $0 \cdot 2501$ |
| $c_{9}$ | $-0.1568$ | $-0.0838$ | $-0.0240$ | 0.0963 | $-0.3120$ | $0 \cdot 3208$ |
| $c_{10}$ | $0 \cdot 4106$ | 0.2973 | -0.2490 | $0 \cdot 1308$ | $-0.2758$ | $0 \cdot 2356$ |
| $c_{20}$ | $0 \cdot 2538$ | -0.2558 | $-0.0109$ | $-0.1260$ | $0 \cdot 1807$ | $0 \cdot 2022$ |
| $c_{21}$ | $-0.5076$ | $-0.2332$ | -0.0438 | $0 \cdot 1110$ | $-0.2928$ | $0 \cdot 2717$ |

Table 5. Charge densities in chlorin.

| Position | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 20 | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $q_{i}$ | $\ldots \ldots$ | 1.1117 | 1.0428 | 1.0996 | 1.0771 | 1.0968 | 1.0947 | 1.0802 | 1.0046 | 1.0786 | 1.0915 | 1.2921 |

$\mathrm{C}_{2 \mathrm{v}}$ Tetrahydroporphin.-As in chlorin the M.O. wave functions are bases of the irreducible representations $A_{2}$ and $B_{1}$. A further simplification can be made as the $A_{2}$ wave functions are bases of the irreducible representations $\mathrm{A}_{2}{ }^{\prime}$ and $\mathrm{B}_{1}{ }^{\prime}$ of the group corresponding to $\mathrm{A}_{2}$ which is of

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course isomorphous with $\mathrm{C}_{2 \mathrm{r}}$. In this way the ninth-order secular determinant of $\mathrm{A}_{2}$ factorises seventh- and second-order determinants. Table 6 lists the roots of the secular equations and the corresponding coefficients. The charge densities are given in Table 7.

Table 6. Values of $x_{r}$ and $c_{i r}$ for $\mathrm{C}_{2 \mathrm{v}}$ tetrahydroporphin.

|  | $\underbrace{\mathrm{A}_{2}}$ |  | $\mathrm{B}_{1}{ }^{\prime}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Represen | tion $\mathrm{A}_{2}$. |  |  |  |  |
| $x_{r}$ | -1.6180 | 0.6180 | $-2 \cdot 0000$ | -1.4812 | $-0.6180$ | 0.3111 | 1.0000 | 1.6180 | $2 \cdot 1701$ |
| $c_{1} \ldots \ldots$ | $0 \cdot 0000$ | $0 \cdot 0000$ | $0 \cdot 0000$ | $0 \cdot 0000$ | $0 \cdot 0000$ | $0 \cdot 0000$ | $0 \cdot 0000$ | $0 \cdot 0000$ | $0 \cdot 0000$ |
| $c_{2} \ldots \ldots$. | $0 \cdot 0000$ | $0 \cdot 0000$ | 0.0816 | $0 \cdot 2779$ | $0 \cdot 3804$ | $0 \cdot 3621$ | $0 \cdot 2887$ | 0.2351 | $0 \cdot 0408$ |
| $c_{3} \ldots$ | $0 \cdot 0000$ | $0 \cdot 0000$ | $-0 \cdot 1633$ | $-0.4117$ | $-0.2351$ | $0 \cdot 1126$ | 0.2887 | $0 \cdot 3804$ | $0 \cdot 0886$ |
| $c_{4}$ | $0 \cdot 0000$ | $0 \cdot 0000$ | $0 \cdot 2449$ | $0 \cdot 3318$ | -0.2351 | $-0.3270$ | $0 \cdot 0000$ | $0 \cdot 3804$ | $0 \cdot 1515$ |
| $c_{5}$. | $0 \cdot 0000$ | $0 \cdot 0000$ | $-0.3266$ | -0.0798 | $0 \cdot 3804$ | -0.2144 | -0.2887 | $0 \cdot 2351$ | $0 \cdot 2401$ |
| $c_{6}$. | $0 \cdot 0000$ | $0 \cdot 0000$ | $0 \cdot 4082$ | -0.2136 | $0 \cdot 0000$ | $0 \cdot 2603$ | -0.2887 | $0 \cdot 0000$ | $0 \cdot 3696$ |
| $c_{7} \ldots \ldots$. | $0 \cdot 2629$ | $0 \cdot 4254$ | -0.2449 | $0 \cdot 1981$ | $-0.1902$ | $0 \cdot 1477$ | $0 \cdot 0000$ | $-0.1176$ | $0 \cdot 2810$ |
| $c_{8} \ldots \ldots$. | $0 \cdot 4254$ | $0 \cdot 2629$ | 0.0816 | $-0.0798$ | $0 \cdot 1176$ | $-0.2144$ | $0 \cdot 2887$ | $-0.1902$ | $0 \cdot 2401$ |
| $c_{9} \ldots \ldots$. | -0.4254 | $-0.2629$ | 0.0816 | -0.0798 | $0 \cdot 1176$ | -0.2144 | $0 \cdot 2887$ | -0.1902 | $0 \cdot 2401$ |
| $c_{10}$ | $0 \cdot 0000$ | $0 \cdot 0000$ | $0 \cdot 0000$ | $0 \cdot 0000$ | $0 \cdot 0000$ | $0 \cdot 0000$ | $0 \cdot 0000$ | $0 \cdot 0000$ | $0 \cdot 0000$ |
| $c_{19}$. | -0.2629 | $-0 \cdot 4254$ | $-0.2449$ | $0 \cdot 1981$ | -0.1902 | $0 \cdot 1477$ | $0 \cdot 0000$ | $-0.1176$ | $0 \cdot 2810$ |


| $x_{r} \ldots \ldots$. | $-2 \cdot 1242$ | -2.0000 | -1.7855 | -1.0977 | $-0.4303$ | $0 \cdot 0000$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $c_{1} \ldots \ldots$. | $0 \cdot 0801$ | 0.2357 | $0 \cdot 4148$ | $0 \cdot 3974$ | 0.2243 | 0.3536 |
| $c_{2} \ldots \ldots$. | -0.0851 | -0.2357 | $-0.3703$ | $-0.2181$ | $-0.0483$ | $0 \cdot 0000$ |
| $c_{3} \ldots \ldots$. | $0 \cdot 1007$ | $0 \cdot 2357$ | $0 \cdot 2464$ | -0.1580 | $-0.2035$ | $-0.3536$ |
| $c_{4} \ldots \ldots$. | $-0.1287$ | $-0.2357$ | -0.0696 | $0 \cdot 3916$ | $0 \cdot 1361$ | $0 \cdot 0000$ |
| $c_{5} \ldots \ldots$. | $0 \cdot 1727$ | $0 \cdot 2357$ | -0.1221 | $-0.2718$ | $0 \cdot 1450$ | $0 \cdot 3536$ |
| $c_{6} \ldots \ldots$. | $-0.2382$ | $-0.2357$ | $0 \cdot 2876$ | $-0.0932$ | $-0 \cdot 1983$ | $0 \cdot 0000$ |
| $c_{7} \ldots \ldots$. | $0 \cdot 0261$ | $0 \cdot 2357$ | $-0.2539$ | $0 \cdot 2410$ | -0.2484 | $0 \cdot 0000$ |
| $c_{8} \ldots \ldots$. | $0 \cdot 1828$ | $-0.2357$ | $0 \cdot 1658$ | $-0.1714$ | $0 \cdot 3052$ | 0.0000 |
| $c_{9} \ldots \ldots$. | $-0.4143$ | $0 \cdot 2357$ | -0.0421 | $-0.0529$ | $0 \cdot 1171$ | 0.0000 |
| $c_{10}$. | $0 \cdot 3901$ | $-0.2357$ | 0.0472 | $0 \cdot 0964$ | 0.5443 | 0.3536 |
| $c_{19} \quad \cdots$ | $0 \cdot 3072$ | $0 \cdot 0000$ | $-0.1375$ | 0.1331 | $0 \cdot 1886$ | $-0.3536$ |
| $x_{r} \ldots \ldots$. | $0 \cdot 6495$ | $1 \cdot 0000$ | 1.5345 | 1.9191 | $2 \cdot 3346$ |  |
| $c_{1} \ldots \ldots$. | 0.2187 | $0 \cdot 3333$ | $0 \cdot 3255$ | $0 \cdot 4082$ | 0.0333 |  |
| $c_{2} \ldots \ldots$. | $0 \cdot 0710$ | $0 \cdot 1667$ | $0 \cdot 2498$ | $0 \cdot 3916$ | $0 \cdot 0389$ |  |
| $c_{3} \ldots \ldots$. | -0.1725 | $-0.1667$ | 0.0577 | $0 \cdot 3435$ | 0.0575 |  |
| $c_{4} \ldots \ldots$. | $-0.1831$ | $-0.3333$ | -0.1612 | $0 \cdot 2675$ | $0 \cdot 0953$ |  |
| $c_{5} \ldots \ldots$. | 0.0563 | -0.1667 | $-0.3050$ | $0 \cdot 1699$ | $0 \cdot 1649$ |  |
| $c_{6} \ldots \ldots$. | $0 \cdot 2179$ | $0 \cdot 1667$ | $-0.3069$ | 0.0585 | $0 \cdot 2898$ |  |
| $c_{7} \ldots \ldots$ | $-0.2703$ | 0.3333 | -0.1449 | $-0.0124$ | $0 \cdot 2331$ |  |
| $c_{8} \ldots \ldots$ | $-0.3935$ | $0 \cdot 1667$ | $0 \cdot 0846$ | $-0.0822$ | 0.2543 |  |
| $c_{9} \ldots \ldots$ | $0 \cdot 0148$ | $-0.1667$ | 0.2747 | $-0.1453$ | 0.3606 |  |
| $c_{10} \ldots$ | $0 \cdot 0454$ | $-0.3333$ | $0 \cdot 3580$ | $-0.1515$ | $0 \cdot 3089$ |  |
| $c_{19} \ldots$ | $0 \cdot 3582$ | $0 \cdot 0000$ | $-0.0210$ | $-0.0453$ | $0 \cdot 2786$ |  |

Representation $\mathrm{B}_{\mathbf{1}}$.

Table 7. Charge densities in $\mathrm{C}_{2 \mathrm{v}}$ tetrahydroporphin.

| Position | $\ldots$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $q_{1} \ldots \ldots . .$. | $1 \cdot 1153$ | 1.0429 | 1.1115 | 1.0517 | 1.0946 | 1.0892 | 1.1049 | 1.1068 | 1.0937 | 0.9694 | 1.2578 |

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[^0]:    ${ }^{1}$ Cf. Stoll and Wiedemann, Fortschr. chem. Forsch., 1952, 2, 538 and references there cited.

[^1]:    ${ }^{3}$ Mittenzwei, ibid., 1942, 275, 93.
    ${ }^{4}$ Ficken, Johns, and Linstead, Chem. Soc. Spec. Publ. No. 3, 1955.
    ${ }^{5}$ Seely, U.S. Atomic Energy Commn. U.C.R.L. 2417, 1953.

[^2]:    * This symmetry is partially destroyed by the presence of the two pyrrolic hydrogen atoms but the assumption of $D_{4 \mathrm{~h}}$ symmetry is a reasonable approximation.
    ${ }^{6}$ Longuet-Higgins, Rector, and Platt, J. Chem. Phys., 1950, 18, 1174.
    ${ }_{8}^{7}$ See Platt, ibid., p. 1168, for a qualitative discussion of configuration interaction.
    ${ }^{8}$ Cf. Davies, Trans. Faraday Soc., 1955, 51, 449.
    ${ }_{10}^{9}$ Coulson and Longuet-Higgins, Proc. Roy. Soc., 1947, A, 191, 39.
    ${ }^{10}$ Eisner and Linstead, J., 1955, 3742.

[^3]:    * The proof that these compounds are actually at the tetrahydro- rather than the hexahydro-level is still required, although Seely ${ }^{5}$ has some evidence for the separate existence of the hexahydro-derivative which, if correct, locates the longest-wavelength band at $6420 \AA$ for the zinc derivative.

[^4]:    12 Brown, Quart. Rev., 1952, 6, 63.
    13 Wheland, J. Amer. Chem. Soc., 1941, 63, 2025.
    14 Idem, ibid., 1942, 64, 900.

[^5]:    15 Matlow, J. Chem. Phys., 1955, 23, 673.

